## Types of Binary Stars (the majority of stars may be in binaries)

Optical doubles - Two stars in nearly the same line-of-sight, but at very different distances. Not true binaries.

Gravitationally associated systems:
Visual binaries - can resolve the 2 stars --> stars nearby or widely separated.
There are $>64,000$ such systems.


Astrometric binaries - Can't resolve the pair, but observe 'wavy' motion on the sky.


Spectroscopic binaries - unresolved, no wavy motion, radial velocity changes in the spectra.

Spectrum binaries - None of the above characteristics, but odd spectrum.

Eclipsing binaries - Edge-on, magnitudes change with time. Thousands known.

## Visual Binaries and Stellar Mass Determinations

Motion on the sky of a visual binary -


Where the arrow gives the mean motion (proper motion in "/yr), and both stars are observed.

If the orbital period is $\leq 100 \mathrm{yrs}$, then we can observe at least one whole period. Subtract out the mean motion, then derive the apparent relative orbit by assuming that the fainter star revolves around the brighter star. (This is merely a conceptual convenience, useful for the moment.)

The next complication is in figuring out the tilt or inclination of the binary orbit relative to the plane of the sky. Generally, the apparent relative orbit will be an ellipse. (Of course, the true orbit of the secondary star about the center-of-mass is also an ellipse, but a different ellipse.) The primary star should be at one focus of the ellipse, if the orbit were not titlted.

If the orbit is tilted, the line connecting the primary to the ellipse center will not generally coincide with the apparent semi-major axis. The angle $\theta$ depends on the inclination.


Computer programs can then take the data and 'de-tilt' until $\theta \approx 0$. Then we can derive the true angular semi-major axis.

This + a distance estimate --> semi-major axis a.
Combine this with the observed period, P, and Kepler's 3rd Law to get,

$$
\frac{M_{1}+M_{2}}{M_{s u n}}=a^{3}(a u) / P^{2}(y r)
$$

i.e., the total mass in the binary system.

Now use the center-of-mass 'teeter-totter' law: $\quad \mathrm{r}_{1} / \mathrm{r}_{2}=\mathrm{M}_{2} / \mathrm{M}_{1}$.

The apparent angular values of $r_{1}, r_{2}$ equal the maximum excursions of each star from the mean motion line on the sky. The inclination divides out of the ratio $r_{1} / r_{2}$.

Can now solve for $\mathrm{M}_{1}, \mathrm{M}_{2}$.

Example: Sirius A, B.
$\mathrm{a} "=7.5 ", \mathrm{~d}=2.67 \mathrm{pc}, \mathrm{P}=50 \mathrm{yr}$.
$\mathrm{a}(\mathrm{au})=7.5 \times 2.67=20.0 \quad$ (parallax angle $=$ triangle base $/$ side,$\pi "=1 \mathrm{au} / \mathrm{d}(\mathrm{pc})$.
---> $d(p c)=a(a u) / a ")$

Then, $\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}=20.0^{3} / 50^{2}=3.2$ solar masses.

Also, $\mathrm{r}_{\mathrm{B}} / \mathrm{r}_{\mathrm{A}} \approx 2=\mathrm{M}_{\mathrm{A}} / \mathrm{M}_{\mathrm{B}} . \quad-->\quad \mathrm{M}_{\mathrm{A}} \approx 2 \mathrm{M}_{\mathrm{B}}$,

Therefore, $\mathrm{M}_{\mathrm{B}} \approx 1 \mathrm{M}_{\text {sun }}, \quad \mathrm{M}_{\mathrm{A}} \approx 2 \mathrm{M}_{\text {sun }}$.

This is the direct method for determining stellar masses.

## Summary:

Measure: $\mathrm{P}, \theta, \mathrm{r}_{1}{ }^{\prime}, \mathrm{r}_{2}{ }^{\prime}$, d.
Derive: $\mathrm{a}, \mathrm{M}_{1}, \mathrm{M}_{2}$, and the true orbit.

## Astrometric Binaries

Single 'wavy' curve on the sky
---> can measure P, but not the separation.

## Spectroscopic Binaries

We can't observe their motions on the sky, so move to velocity space. Observe periodic Doppler shifts. If we observe one binary component through a whole period, we see blue shifts when the star is coming towards us, red shifts when its moving away.


We can get the radial velocity curve from many such observations.

$\Delta v$ is a minimum estimate of the true velocity amplitude, since we don't know the orbital inclination.
( $\Delta \mathrm{v}$ is determined from the usual Doppler shift formula.)

## The simplest special case: Circular orbits viewed edge-on.

Sinusoidal radial velocity curves --> circular orbits.

$$
2 \pi r_{1}=v_{1} \mathrm{P}, \quad 2 \pi r_{2}=\mathrm{v}_{2} \mathrm{P} .
$$

Measure $\mathrm{v}_{1}, \mathrm{v}_{2}$, and P . Solve for $\mathrm{r}_{1}, \mathrm{r}_{2}$. Also get the mass ratio:

$$
\frac{m_{1}}{m_{2}}=\frac{r_{2}}{r_{1}}=\frac{v_{2}}{v_{1}}
$$

Use Kepler's 3rd law with $\mathrm{r}_{1}+\mathrm{r}_{2}=\mathrm{a}$,

$$
\mathrm{m}_{1}+\mathrm{m}_{2}=\mathrm{a}^{3} / \mathrm{P}^{2} .
$$

This is just like the procedure for visual binaries, except $\mathrm{v}_{1}, \mathrm{v}_{2}$ are observed instead of $r_{1}$ ", $r_{2}$ ".

If the relative orbit is inclined by angle $i$, then the observed velocities are related to the true velocities, $\mathrm{v}_{1,2}$, by -

$$
\mathrm{v}_{1,2}^{\prime}=\mathrm{v}_{1,2} \sin (\mathrm{i}) \quad \text { (like } \mathrm{r}_{1,2}^{\prime}=\mathrm{r}_{1,2} \sin (\mathrm{i}) \text { for visual binaries) }
$$

- Can still deduce the mass ratio

$$
\frac{m_{1}}{m_{2}}=\frac{v_{2}}{v_{1}}=\frac{v_{2}}{v_{2}^{\prime}}
$$

- The period is unchanged as well.
- But no longer know the semi-major axis a, only $a^{\prime}=$ a $\sin (i)$.

Kepler's 3rd becomes -

$$
m_{1}+m_{2}=\frac{a^{\prime 3}}{P^{2} \sin ^{3} i}
$$

However, in this case we cannot determine $\sin (\mathrm{i})$. In fact, we cannot even determine $a^{\prime}$, only $r_{1}{ }^{\prime}$. In terms of this observable...

$$
a^{\prime}=r_{1}^{\prime}+r_{2}^{\prime}=r_{1}^{\prime}\left(1+\frac{r_{2}^{\prime}}{r_{1}^{\prime}}\right)=r_{1}^{\prime}\left(1+\frac{m_{1}}{m_{2}}\right)
$$

Substituting into Kepler's 3rd, we get,

$$
\frac{r_{1}^{\prime 3}}{P^{2}}=\frac{\left(m_{1}+m_{2}\right) \sin ^{3}(i)}{\left(1+\frac{m_{1}}{m_{2}}\right)^{3}}=\frac{m_{2}^{3} \sin ^{3}(i)}{\left(m_{1}+m_{2}\right)^{2}}=f\left(m_{1}, m_{2}\right)
$$

This is called the mass function.
We may also have an estimate for $\mathrm{m}_{1}$ from the star's spectral type. Then, $\mathrm{f}\left(\mathrm{m}_{1}, \mathrm{~m}_{2}\right)$ constrains the possible values of $\mathrm{m}_{2}$.

Aside from gathering more precision data on stellar masses in general, this is also very important for exotic systems. E.g., black hole candidate systems.

MCl initecte (IMV)
nole:


LMC $X-3$


$$
\begin{gathered}
A 0620-00 \\
M_{x}=13 M_{\odot} \overbrace{0}^{M_{c}=0.7 M_{\odot}}
\end{gathered}
$$

TABLE 1
EROPERTIES OF THREE BLACK-HOLE BINARIES

|  | CYg $\mathrm{X}-1$ | LMC $x-3$ | A0620-00 |
| :---: | :---: | :---: | :---: |
| $L_{\text {x }}$ (ergs $\mathrm{s}^{\text {si }}$ ) | $2 \times 10^{37}$ | $3 \times 10^{20}$ | 1×10* |
| MK type | 09.71 ab | B3V | KSV |
| d (kpe) | 2.5 | 55 | 1 |
| $\square_{0}$ | ? | 17 | 18 |
| $V_{e} \sin 1\left(k m s^{-i}\right)$ | $76 \pm 1$ | $235 \pm 11$ | $457 \pm 8$ |
| P (days) | 5.6 | 1.7 | 0.32 |
| $\leq(\mathrm{M} / \mathrm{M})$ | $0.25 \doteqdot 0.01$ | $2.3=0.3$ | $3.18 \div 0.16$ |

ESO Messenger 126, 29 (2006) Kaper et al.

Figure 2: Neutron-star and black-hole masses obtained from the literature (Stairs 2004, McClintock and Remillard 2005, and references therein). The
neutron stars, especially the binary radio pulsars (at the bottom), occupy a relatively narrow mass range near $1.35 \mathrm{M}_{\odot}$. The X -ray pulsars (to the middle)
show a wider spread, including two systems with a neutron-star mass near $2 \mathrm{M}_{\odot}$. Such a high neutronstar mass would rule out a soft equation of state. The black-hole candidates (at the top) are significantly more massive, indicative of a different formation mechanism


## Astronomy Picture of the Day

Discover the cosmos! Each day a different image or photograph of our fascinating universe is featured, along with a brief explanation written by a professional astronomer.


X-Rays Indicate Star Ripped Up by Black Hole Illustration Credit: M. Weiss, CXC, NASA

Explanation: What could rip a star apart? A black hole. Giant black holes in just the right mass range would pull on the front of a closely passing star much more strongly than on the back. Such a strong tidal force would stretch out a star and likely cause some of the star's gasses to fall into the black hole. The infalling gas has been predicted to emit just the same blast of X-rays that have recently been seen in the center of galaxy RX J1242-11. Above, an artist's illustration depicts the sequence of destruction (assuming that image-distorting gravitational-lens effects of the black hole are somehow turned off). Most of the stellar remains would be flung out into the galaxy. Such events are rare, occurring perhaps only one in 10,000 years for typical black holes at the center of typical galaxies.

Tomorrow's picture: boat the rock

Some final bits on light curves.

222 CHAPTER 12 STARS: BINARY SYSTEMS




B



A


Brightness


C

