Harmony & Frequency

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Purpose

If pitches are sound waves with a different frequencies, then how are the frequencies determined so that a person playing an 'A' in California is playing the same pitched 'A' in South Carolina?

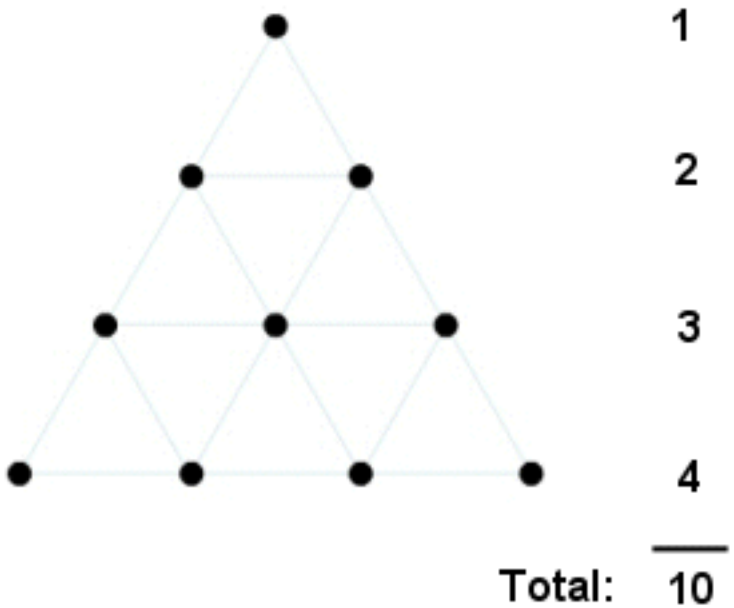
Pythagoras

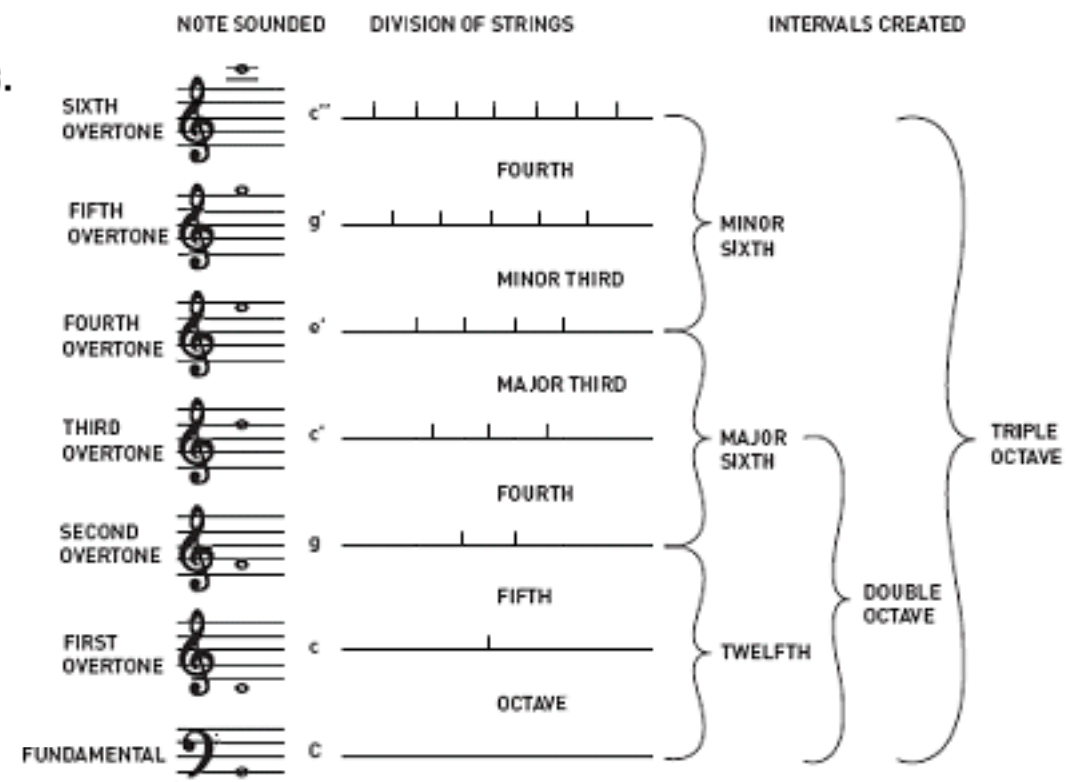
There is geometry in the humming of the strings
... there is music in the spacing of the spheres.

Pythagoras

- Believed the universe, religion, and mathematics worked together harmoniously.
- *Musica Universalis* "Music of the Spheres"
- Using a 'single-stringed guitar or harp' (very primitive form) he was able to explain his theories mathematically, visually, and aurally.

The tetractys, an equilateral triangular figure consisting of 10 points arranged in four rows of 1, 2, 3 and 4, was both a mathematical idea and a metaphysical symbol for the Pythagoreans.





Pythagoras' Findings

Tuning Systems

- Even though music has evolved greatly since the time of Pythagoras, it is not absurd to argue that the method of tuning (devising the frequencies for each pitch) is not too dissimilar.
- Pythagoras = Perfect Consonances (perfect ratios between pitches)

Tuning Systems: Just Intonation

- Using simple numeric ratios to determine distances between pitches.
- C 1:1, D 9:8, E 5:4, F 4:3, G 3:2, A 5:3, B 15:8, C 2:1 to define the ratios for the notes in a C major scale.
- Pythagorean system is a form of just intonation.
- Many other forms simple ratios of whole numbers could be used.
- Occurs naturally in strings, bagpipes, and barbershop quartets (due to close harmonies).
- Possible for Trombones to utilize it on the fly.
- Problematic for piano and other fixed instruments.

Tuning Systems: Equal Temperament

- Adjacent pitches have identical frequency ratios. OR
- An octave is divided into 12 equal parts.
- Depending on the number of equal parts or the distance of the octave, many different versions of the equal temperament system existed from 1600 to today.
- The word equal is misleading.   ET is designed to allow some flexibilities from the hard ratios of just intonation.

Name	Exact value in 12-TET	Decimal value in 12-TET	Cents	Just intonation interval	Cents in just intonation	Difference
Unison (C)	$2^{\frac{0}{12}} = 1$	1.000000	0	$\frac{1}{1}$ = 1.000000	0.00	0
Minor second (C♯/D♭)	$2^{\frac{1}{12}} = \sqrt[12]{2}$	1.059463	100	$\frac{16}{15}$ = 1.066667	111.73	11.73
Major second (D)	$2^{\frac{2}{12}} = \sqrt[6]{2}$	1.122462	200	$\frac{9}{8}$ = 1.125000	203.91	3.91
Minor third (D♯/E♭)	$2^{\frac{3}{12}} = \sqrt[4]{2}$	1.189207	300	$\frac{6}{5}$ = 1.200000	315.64	15.64
Major third (E)	$2^{\frac{4}{12}} = \sqrt[3]{2}$	1.259921	400	$\frac{5}{4}$ = 1.250000	386.31	−13.69
Perfect fourth (F)	$2^{\frac{5}{12}} = \sqrt[12]{32}$	1.334840	500	$\frac{4}{3}$ = 1.333333	498.04	−1.96
Augmented fourth (F♯/G♭)	$2^{\frac{6}{12}} = \sqrt{2}$	1.414214	600	$\frac{7}{5}$ = 1.400000	582.51	−17.49
Perfect fifth (G)	$2^{\frac{7}{12}} = \sqrt[12]{128}$	1.498307	700	$\frac{3}{2}$ = 1.500000	701.96	1.96
Minor sixth (G♯/A♭)	$2^{\frac{8}{12}} = \sqrt[3]{4}$	1.587401	800	$\frac{8}{5}$ = 1.600000	813.69	13.69
Major sixth (A)	$2^{\frac{9}{12}} = \sqrt[4]{8}$	1.681793	900	$\frac{5}{3}$ = 1.666667	884.36	−15.64
Minor seventh (A♯/B♭)	$2^{\frac{10}{12}} = \sqrt[6]{32}$	1.781797	1000	$\frac{7}{4}$ = 1.750000	968.83	−31.17
Major seventh (B)	$2^{\frac{11}{12}} = \sqrt[12]{2048}$	1.887749	1100	$\frac{15}{8}$ = 1.875000	1088.27	−11.73
Octave (C)	$2^{\frac{12}{12}} = 2$	2.000000	1200	$\frac{2}{1}$ = 2.000000	1200.00	0

Just Intonation vs. Equal Temperament

Comparison

http://www.youtube.com/watch?v=BhZpvGSPx6w