Review Modern Physics for Exam 1

Time Dilation, Length Contraction

2. (I) A certain type of elementary particle travels at a speed of 2.70 $\times 10^{8} m/s$ .At this speed, the average lifetime is measured to be 4.76 $\times 10^{-6}$s. What is the particle's lifetime at rest?

We find the lifetime at rest

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} = \left(4.76\times10^{-6}\,\text{s}\right)\sqrt{1-\left(\frac{2.70\times10^{8}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2} = \boxed{2.07\times10^{-6}\text{s}}$$

4. (II) If you were to travel to a star 135 light-years from Earth at a speed of 2.80 $\times 10^{8} m/s,$ what would you measure this distance to be?

The measured distance is the contracted length.

$$l = l_0\sqrt{1-v^2/c^2} = \left(135\text{ ly}\right)\sqrt{1-\left(\frac{2.80\times10^{8}\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2} = \boxed{48.5\text{ ly}}$$

5. (II) What is the speed of a pion if its average lifetime is measured to be 4.40 $\times 10^{-8} s$. At rest, its average lifetime is 2.60 $\times 10^{-8} s$

The speed is determined from the time dilation relationship,

$$\Delta t_0 = \Delta t\sqrt{1-v^2/c^2} \quad \rightarrow$$

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{2.60\times10^{-8}\text{s}}{4.40\times10^{-8}\text{s}}\right)^2} = \boxed{0.807c} = 2.42\times10^{8}\text{ m/s}$$

6. (II) In an Earth reference frame, a star is 56 light-years away. How fast would you have to travel so that to you the distance would be only 35 light-years?

The speed is determined from the length contraction relationship,

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad\rightarrow\quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-\left(\frac{35\,\text{ly}}{56\,\text{ly}}\right)^2} = \boxed{0.78c} = 2.3\times10^8\ \text{m/s}$$

7. (II) Suppose you decide to travel to a star 65 light-years away at a speed that tells you the distance is only 25 light-years. How many years would it take you to make the trip?

The speed is determined from the length contraction relationship,
Then the time is found from the speed and the contracted distance.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad\rightarrow$$

$$v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}\ ;\ t = \frac{\ell}{v} = \frac{\ell}{c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2}} = \frac{25\,\text{ly}}{c\sqrt{1-\left(\frac{25\,\text{ly}}{65\,\text{ly}}\right)^2}} = \frac{(25\,\text{y})c}{c(0.923)} = \boxed{27\,\text{y}}$$

8. (II) At what speed y will the length of a 1.00-m stick look 10.0% shorter (90.0 cm)?

The speed is determined from the length contraction relationship,

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad\rightarrow\quad v = c\sqrt{1-\left(\frac{\ell}{\ell_0}\right)^2} = c\sqrt{1-(0.900)^2} = \boxed{0.436c}$$

9. (II) Escape velocity from the Earth is $11.2\ \text{km/s}$. What would be the percent decrease in length of a 65.2-m-long spacecraft traveling at that speed as seen from Earth?

The change in length is determined from the length contraction relationship. The speed is very small compared to the speed of light.

$$\ell = \ell_0\sqrt{1-v^2/c^2} \quad\rightarrow$$

$$\frac{\ell}{\ell_0} = \sqrt{1-v^2/c^2} = \left(1-\frac{v^2}{c^2}\right)^{1/2} \approx 1-\tfrac{1}{2}\frac{v^2}{c^2} = 1-\tfrac{1}{2}\left(\frac{11.2\times10^3\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2 = 1-6.97\times10^{-10}$$

So the percent decrease is $\boxed{(6.97 \times 10^{-8})\%}$.

10. (II) A friend speeds by you in her spacecraft at a speed of 0.760*c*. It is measured in your frame to be 4.80 m long and 1.35 m high. (*a*) What will be its length and height at rest? (*b*) How many seconds elapsed on your friend's watch when 20.0 s passed on yours? (*c*) How fast did you appear to be traveling according to your friend? (*d*) How many seconds elapsed on your watch when she saw 20.0 s pass on hers?

a) The measured length is the contracted length. We find the rest length from

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}} = \frac{4.80\,\text{m}}{\sqrt{1 - (0.760)^2}} = \boxed{7.39\,\text{m}}$$

Distances perpendicular to the motion do not change, so the rest height is $\boxed{1.35\,\text{m}}$.

(*b*) The time in the spacecraft is the rest time, found from

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (20.0\,\text{s})\sqrt{1 - (0.760)^2} = \boxed{13.0\,\text{s}}$$

(*c*) To your friend, you moved at the same relative speed: $\boxed{0.760c}$.

(*d*) She would measure the same time dilation: $\boxed{13.0\,\text{s}}$.

11. (II) At what speed do the relativistic formulas for (*a*) length and (*b*) time intervals differ from classical values by 1.00%? (This is a reasonable way to estimate when to do relativistic calculations rather than classical.)

a) We use the equation for length contraction with the contracted length 99.0% of the rest length.

$$l = l_0\sqrt{1 - v^2/c^2} \quad \to \quad v = c\sqrt{1 - \left(\frac{l}{l_0}\right)^2} = c\sqrt{1 - (0.990)^2} = \boxed{0.141c}$$

(*b*) for time dilation with the time as measured from a relative moving frame 1.00% greater than the rest time.

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} \quad \to \quad v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1 - \left(\frac{1}{1.0100}\right)^2} = \boxed{0.140c}$$

We see that a speed of 0.14 *c* results in about a 1% relativistic effect.

12. (II) A certain star is 18.6 light-years away. How long would it take a spacecraft traveling 0.950*c* to reach that star from Earth, as measured by observers: (*a*) on Earth, (*b*) on the spacecraft? (*c*) What is the distance traveled according to observers on the spacecraft? (*d*) What will the spacecraft occupants compute their speed to be from the results of (*b*) and (*c*)?

12. (*a*) To an observer on Earth, 18.6 ly is the rest length, so the time will be the distance divided by
the speed.

$$t_{\text{Earth}} = \frac{l_0}{v} = \frac{(18.6\text{ ly})}{0.950c} = 19.58\text{ yr} \approx \boxed{19.6\text{ yr}}$$

(*b*) The time as observed on the spacecraft is shorter. Use

$$\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (19.58\text{ yr})\sqrt{1-(0.950)^2} = 6.114\text{ yr} \approx \boxed{6.11\text{ yr}}$$

(*c*) To the spacecraft observer, the distance to the star is contracted. Use

$$l = l_0\sqrt{1 - v^2/c^2} = (18.6\text{ ly})\sqrt{1-(0.950)^2} = 5.808\text{ ly} \approx \boxed{5.81\text{ ly}}$$

(*d*) To the spacecraft observer, the speed of the spacecraft is their observed distance divided by
their observed time.

$$v = \frac{l}{\Delta t_0} = \frac{(5.808\text{ ly})}{6.114\text{ yr}} = \boxed{0.950c}$$

Lorentz Transformations

23. (II) Two spaceships leave Earth in opposite directions, each with a speed of 0.60*c* with respect to Earth. (*a*) What is the velocity of spaceship 1 relative to spaceship 2? (*b*) What is the velocity of spaceship 2 relative to spaceship 1?

(*a*) We take the positive direction to be the direction of motion of spaceship 1. Consider spaceship
2 as reference frame S, and the Earth reference frame S′. The velocity of the Earth relative to spaceship 2 is $v = 0.60c$. The velocity of spaceship 1 relative to the Earth is $u'_x = 0.60c$. Solve for the velocity of spaceship 1 relative to spaceship 2, u_x,

$$u_x = \frac{(u'_x + v)}{\left(1 + \frac{vu'_x}{c^2}\right)} = \frac{(0.60c + 0.60c)}{[1 + (0.60)(0.60)]} = \boxed{0.88c}$$

(*b*) Now consider spaceship 1 as reference frame S. The velocity of the Earth relative to spaceship
1 is $v = -0.60c$. The velocity of spaceship 2 relative to the Earth is $u'_x = -0.60c$. Solve for the velocity of spaceship 2 relative to spaceship 1, u_x, using

$$u_x = \frac{(u_x' + v)}{\left(1 + \frac{vu_x'}{c^2}\right)} = \frac{(-0.60c - 0.60c)}{[1 + (-0.60)(-0.60)]} = \boxed{-0.88c}$$

As expected, the two relative velocities are the opposite of each other.

25. (II) A spaceship leaves Earth traveling at $0.61c$. A second spaceship leaves the first at a speed of $0.87c$ with respect to the first. Calculate the speed of the second ship with respect to Earth if it is fired (*a*) in the same direction the first spaceship is already moving, (*b*) directly backward toward Earth.

(*a*) We take the positive direction in the direction of the first spaceship. We choose reference frame S as the Earth, and reference frame S' as the first spaceship. So $v = 0.61c$. The speed of the second spaceship relative to the first spaceship is $u_x' = 0.87c$. We use Lorentz equations to solve for the speed of the second spaceship relative to the Earth, u.

$$u_x = \frac{(u_x' + v)}{\left(1 + \frac{vu_x'}{c^2}\right)} = \frac{(0.87c + 0.61c)}{[1 + (0.61)(0.87)]} = \boxed{0.97c}$$

(*b*) The only difference is now that $u_x' = -0.87c$.

$$u_x = \frac{(u_x' + v)}{\left(1 + \frac{vu_x'}{c^2}\right)} = \frac{(-0.87c + 0.61c)}{[1 + (0.61)(-0.87)]} = -0.55c$$

The problem asks for the speed, which would be $\boxed{0.55c}$

26. (II) Your spaceship, traveling at $0.90c$, needs to launch a probe out the forward hatch so that its speed relative to the planet that you are approaching is $0.95c$. With what speed must it leave your ship?

We assume that the given speed of $0.90c$ is relative to the planet that you are approaching. We take the positive direction in the direction that you are traveling. Consider your spaceship as reference frame S, and the planet as reference frame S'. The velocity of the planet relative to you is $v = -0.90c$. The velocity of the probe relative to the planet is $u_x' = 0.95c$. Solve for the velocity of the probe relative to your spaceship, u_x,

$$u_x = \frac{(u_x' + v)}{\left(1 + \frac{vu_x'}{c^2}\right)} = \frac{(0.95c - 0.90c)}{[1 + (-0.90)(0.95)]} = \boxed{0.34c}$$

27. (II) A spaceship traveling at 0.76*c* away from Earth fires a module with a speed of 0.82*c* at right angles to its own direction of travel (as seen by the spaceship). What is the speed of the module, and its direction of travel (relative to the spaceship's direction), as seen by an observer on Earth?

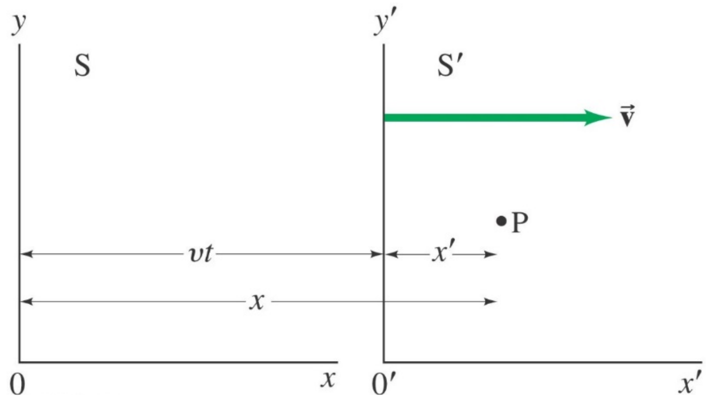
27. We set frame S′ as the frame at rest with the spaceship. In this frame the module has speed $u' = u_y' = 0.82c$. Frame S is the frame that is stationary with respect to the Earth. The spaceship, and therefore frame S′ moves in the *x*-direction with speed 0.76c in this frame, or $v = 0.76c$. We use the Lorentz equations for velocities to determine the components of the module velocity in frame S. Then using trigonometry we combine the components to determine the speed and direction of travel.

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{0 + 0.76c}{1 + 0} = 0.76c \;;\; u_y = \frac{u_y'\sqrt{1 - v^2/c^2}}{1 + vu_x'/c^2} = \frac{0.82c\sqrt{1 - 0.76^2}}{1 + 0} = 0.533c$$

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{(0.76c)^2 + (0.533c)^2} = \boxed{0.93c} \;;\; \theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\frac{0.533c}{0.76c} = \boxed{35°}$$

28. (II) If a particle moves in the *xy* plane of system S (Fig. below) with speed *u* in a direction that makes an angle θ with the *x* axis, show that it makes an angle θ' in S' given by

$$tan\theta' = (sin\theta\sqrt{1 - \frac{v^2}{c^2}}\;/(cos\theta - v/u)$$



28. The velocity components of the particle in the S frame are $u_x = u\cos\theta$ and $u_y = u\sin\theta$. We find the components of the particle in the S' frame from the velocity transformations. Those transformations are for the S' frame moving with speed *v* relative to the S frame. We can find the transformations from the S frame to the S' frame by simply changing *v* to –*v* and primed to unprimed variables.

$$u_x = \frac{(u'_x + v)}{(1 + vu'_x/c^2)} \rightarrow u'_x = \frac{(u_x - v)}{(1 - vu_x/c^2)} \;;\; u_y = \frac{u'_y\sqrt{1 - v^2/c^2}}{(1 + vu_x/c^2)} \rightarrow u'_y = \frac{u_y\sqrt{1 - v^2/c^2}}{(1 - vu_x/c^2)}$$

$$\tan\theta' = \frac{u'_y}{u'_x} = \frac{\dfrac{u_y\sqrt{1 - v^2/c^2}}{(1 - vu_x/c^2)}}{\dfrac{(u_x - v)}{(1 - vu_x/c^2)}} = \frac{u_y\sqrt{1 - v^2/c^2}}{(u_x - v)} = \frac{u\sin\theta\sqrt{1 - v^2/c^2}}{(u\cos\theta - v)} = \boxed{\frac{\sin\theta\sqrt{1 - v^2/c^2}}{(\cos\theta - v/u)}}$$

Relativistic Momentum

34. (I) What is the momentum of a proton traveling at $v = 0.75c$?

34. The momentum of the proton is given by

$$p = \gamma mv = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{(1.67 \times 10^{-27}\ \text{kg})(0.75)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - (0.75)^2}} = \boxed{5.7 \times 10^{-19}\ \text{kg}\cdot\text{m/s}}$$

35. (II) (*a*) A particle travels at $y = 0.10c$. By what percentage will a calculation of its momentum be wrong if you use the classical formula? (*b*) Repeat for $y = 0.60c$.

35. (*a*) We compare the classical momentum to the relativistic momentum.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\dfrac{mv}{\sqrt{1-v^2/c^2}}} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.10)^2} = 0.995$$

The classical momentum is about $\boxed{-0.5\%}$ in error.

(*b*) We again compare the two momenta.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\dfrac{mv}{\sqrt{1-v^2/c^2}}} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.60)^2} = 0.8$$

The classical momentum is $\boxed{-20\%}$ in error.

36. (II) A particle of mass *m* travels at a speed $y = 0.26c$. At what speed will its momentum be doubled?

36. The momentum at the higher speed is to be twice the initial momentum. We designate the initial state with a subscript "0", and the final state with a subscript "f".

$$\frac{p_{\text{f}}}{p_0} = \frac{\dfrac{mv_{\text{f}}}{\sqrt{1-v_{\text{f}}^2/c^2}}}{\dfrac{mv_0}{\sqrt{1-v_0^2/c^2}}} = 2 \rightarrow \frac{\dfrac{v_{\text{f}}^2}{1-v_{\text{f}}^2/c^2}}{\dfrac{v_0^2}{1-v_0^2/c^2}} = 4 \rightarrow \left(\frac{v_{\text{f}}^2}{1-v_{\text{f}}^2/c^2}\right) = 4\left[\frac{(0.26c)^2}{1-(0.26)^2}\right] = 0.29c^2 \rightarrow$$

$$v_{\text{f}}^2 = \left(\frac{0.29}{1.29}\right)c^2 \rightarrow v_{\text{f}} = \boxed{0.47c}$$

37. (II) An unstable particle is at rest and suddenly decays into two fragments. No external forces act on the particle or its fragments. One of the fragments has a speed of $0.60c$ and a mass of $1.67\times10^{27}kg$ while the other has a mass of $6.68\times10^{27}kg$ What is the speed of the less massive fragment?

$\boxed{37.}$ The two momenta, as measured in the frame in which the particle was initially at rest, will be equal to each other in magnitude. The lighter particle is designated with a subscript "1", and the heavier particle with a subscript "2".

$$p_1 = p_2 \rightarrow \frac{m_1 v_1}{\sqrt{1 - v_1^2/c^2}} = \frac{m_2 v_2}{\sqrt{1 - v_2^2/c^2}} \rightarrow$$

$$\frac{v_1^2}{\left(1 - v_1^2/c^2\right)} = \left(\frac{m_2}{m_1}\right)^2 \frac{v_2^2}{\left(1 - v_2^2/c^2\right)} = \left(\frac{6.68\times10^{-27}\,\text{kg}}{1.67\times10^{-27}\,\text{kg}}\right)^2 \left[\frac{(0.60c)^2}{1-(0.60)^2}\right] = 9.0c^2 \rightarrow$$

$$v_1 = \sqrt{0.90}\,c = \boxed{0.95c}$$

38. (II) What is the percent change in momentum of a proton that accelerates (a) from $0.45c$ to $0.80c$, (b) from $0.80c$ to $0.98c$?

38. We find the proton's momenta using

$$p_{0.45} = \frac{m_p v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{m_p(0.45c)}{\sqrt{1-(0.45)^2}} = 0.5039 m_p c \;;\; p_{0.80} = \frac{m_p v_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{m_p(0.80c)}{\sqrt{1-(0.80)^2}} = 1.3333 m_p c$$

$$p_{0.98} = \frac{m_p v_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{m_p(0.98c)}{\sqrt{1-(0.98)^2}} = 4.9247 m_p c$$

(a) $$\left(\frac{p_2 - p_1}{p_1}\right)100 = \left(\frac{1.3333 m_p c - 0.5039 m_p c}{0.5039 m_p c}\right)100 = 164.6 \approx \boxed{160\%}$$

(b) $$\left(\frac{p_2 - p_1}{p_1}\right)100 = \left(\frac{4.9247 m_p c - 1.3333 m_p c}{1.3333 m_p c}\right)100 = 269.4 \approx \boxed{270\%}$$